Fundamental Asymmetries in US Monetary Policymaking: Evidence from a Nonlinear Autoregressive Distributed Lag Quantile Regression Model*

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Abstract

Based on a quantile extension of the nonlinear-ARDL model developed by Shin, Yu and Greenwood-Nimmo (2009), we develop a new approach capable of simultaneously modelling three distinct forms of asymmetry: long-run (reaction) asymmetry, short-run (adjustment) asymmetry and quantile-specific (locational) asymmetry. Applying this model to US monetary policymaking from 1964q2-2008q2 we find that: (i) the reaction function is linear in both output and inflation gaps in the lower quantiles of the interest rate and that the Taylor principle is not upheld; (ii) in the mid quantiles, the reaction function shows a good performance bias and the Taylor principle is observed for positive inflationary shocks only; and (iii.) profound asymmetry characterises the highest quantiles associated with the Volcker Fed.

Keywords: Nonlinear ARDL Model, Quantile Regression, Dynamic Multipliers, Reaction Adjustment and Locational Asymmetries, Asymmetric Central Bank Preferences.

JEL Classifications: C22, C51, E58.

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1 Introduction

The analysis of non-linearities in the reaction function of the central bank is a young but vibrant discipline. We identify three general forms of asymmetry that may characterise monetary policy: reaction asymmetry, adjustment asymmetry and locational asymmetry. The first relates to differential long-run interest rate responses that may be elicited by heterogeneously positive or negative shocks to a given variable. The second describes the differential speed of adjustment of the interest rate to equilibrium under various regimes. The third is associated with the notion that the reaction of the central bank to the inflation and output gaps may depend on the current location of the interest rate within its conditional distribution.

Our goal in this paper is to develop a general framework for asymmetric modelling that integrates each of these three forms of asymmetry in a coherent manner. Essentially, this involves combining established approaches to short-run asymmetric modelling and long-run asymmetric modelling with the quantile regression approach popularised by Koenker and Bassett (1978). However, even before considering quantile regression, combining regime-switching short- and long-run models is likely to be non-trivial when the transition function is not common to both the short- and long-run (Saikkonen, 2008). We approach this issue pragmatically following the asymmetric ARDL approach originated by Shin, Yu and Greenwood-Nimmo (2009, hereafter SYG) which combines adjustment asymmetry with reaction asymmetry subject to a common transition function (in this case we impose a common known threshold value of zero in the construction of partial sum processes). One of the principal benefits of this approach is that, quite unlike the popular Markov-switching or smooth transition models, it is easily estimable by standard OLS. This simplicity renders it an ideal candidate for extension to the case of quantile regression.

We apply both the standard nonlinear ARDL model estimated at the conditional mean of the interest rate distribution (the NARDL-M model) and its quantile extension (the NARDL-Q model) to the analysis of US monetary policy between 1964q2 and 2008q2. In the NARDL-M framework, we are unable to reject the null hypothesis of long-run reaction symmetry with respect to both inflation and output gaps. Furthermore, the null hypothesis of short-run symmetric adjustment cannot be rejected in relation to the output gap. On the basis of these results, one would typically conclude that the Federal Reserve has acted in a linear fashion in the long-run during this time but that its interest rate response to inflationary shocks has been more rapid than in the case of disinflationary shocks. However, the NARDL-Q specification estimated on a range of quantiles reveals pronounced locational asymmetry at higher levels of the interest rate. Our results indicate that the Fed has reacted very cautiously and in a linear fashion when the interest rate is low but that its policy response to both inflation and output gaps has been considerably more aggressive and markedly asymmetric when the interest rate is at higher levels. Hence, our results suggest that the failure to account for locational asymmetry may mask other forms of asymmetry.

By mapping the various quantiles onto the time index, we are able to interpret our results in terms of a time-varying reaction function. We conclude that the Volcker administration engaged in very aggressive and markedly asymmetric monetary policy due to the high level of inflation and its well-documented persistence at the time. Similarly, we conclude that much of Greenspan’s tenure and the early period under Bernanke was characterised by growth-fostering policies in a framework that often acted passively in relation to inflation. During this time, the Fed exploited the manner in which inflation was naturally contained by a range of economic phenomena outside their control, including the globalisation of product markets and wide-ranging labour market reforms. The central quantiles relate mainly to the Burns-Miller era and to the period of transition from Volcker to Greenspan, which were typified by persistent and high rates of inflation in the presence of a fragile economic outlook. Against this backdrop, policy was conducted in an asymmetric manner.
but without the fervour seen during the Volcker disinflation. Many authors have since argued that weak policy prior to Volcker was the root of the economic malaise and that subsequent generations of central bankers have learned from these mistakes. Our results do not entirely support this view. In fact, our results suggest that monetary policy during much of the so-called Great Moderation was considerably weaker than under either Burns or Miller in terms of the magnitude of the policy response to inflationary shocks but that stability was maintained nevertheless due to a sequence of beneficial shocks that served to contain inflation and reduce its persistence.

The paper proceeds in 5 sections. Section 2 reviews the existing literature on asymmetric central bank preferences, asymmetric policy adjustments and locational asymmetries in monetary policymaking. Section 3 introduces the asymmetric ARDL model and its quantile extension and offers a brief discussion of the different forms of asymmetry that can be modelled in this way. Section 4 presents the results of both the NARDL-M and NARDL-Q models of the reaction function of the Federal Reserve between 1964q2 and 2008q2. Section 5 concludes.

2 Asymmetric Monetary Policy

The ubiquitous Taylor (1993) rule models the central bank interest rate decision as a linear function of inflation relative to target (the inflation gap) and output relative to potential (the output gap). Underlying this framework is the assumption that the policymaker strives to minimise a quadratic loss function in the inflation and output gaps. Recently, however, a growing body of literature has promoted the notion that the policy rule may be non-linear and the loss function non-quadratic (Blinder, 1997; Granger and Pesaran, 2000; Cukierman and Muscatelli, 2008). Chief among the reasons for this non-linearity are the propositions that correcting a negative output gap may be more difficult than closing a positive output gap (the ‘pushing on a string’ argument) and that inflation may have a tendency to rise more easily than it falls (the rationale for inflation-hawkism).

Nobay and Peel (2003) demonstrate that the optimal policy solution in a theoretical framework in which policymakers’ preferences are modelled asymmetrically involves both an inflation target and a linear Walsh (1995) contract. They conclude that asymmetric modelling adds realism to the analysis of monetary policy and that it may yield results distinctly inconsistent with the case of quadratic preferences. Furthermore, Siklos and Wohar (2005) extend the authors’ work and argue that the careful construction of asymmetric error-correction models can potentially overcome the problems associated with breaks in the structure of the underlying data. The motivation for the development of asymmetric models is apparent.

An early and notable contribution to the empirical literature was made by Ruge-Murcia (2003). Based on a simple game-theoretic framework in which positive and negative inflationary gaps can be weighted differently by policymakers, the author finds that estimated asymmetric reaction functions for Canada, Sweden and the UK yield results that are quantitatively distinct from those of a symmetric specification. He concludes that asymmetric preferences may explain the negative mean of the inflation gap in these three countries.

Dolado, María-Dolores and Naveira (2005) employ a novel approach in which the loss function of the central bank remains quadratic but the specification of the Phillips curve is nonlinear. They demonstrate that this framework also generates nonlinearity in the reaction function of the central bank. Using the Euler equation approach associated with Clarida, Galí and Gertler (1998) as well as the ordered probit approach suggested by Dolado and Mari-Dolores (2002), the authors find substantial evidence of nonlinearities in Germany, France and Spain but not in the USA. In particular, their results indicate that European central banks have systematically responded more strongly to positive than negative inflation and output gaps. They attribute this finding to labour market rigidities present only in the European countries.
Asymmetric preferences have been widely modelled as threshold effects. Bec, Salem and Collard (2000) use the lagged output gap to determine threshold transitions in a STAR framework and find that the interest rate response to inflation is stronger in a recessionary environment than a boom environment. Martin and Milas (2004) assume that regime transitions are governed by a quadratic logistic function in expected inflation. Using this approach, they find that the Bank of England has pursued an asymmetric policy in which positive inflation gaps attract a more aggressive response than negative gaps. Moreover, their results indicate that the Bank adopted a de facto target band of 1.4% - 2.6% between 1992 and 2000. Bunzel and Enders (2005) estimate a simple threshold model and find that the Greenspan Fed did not respond to inflation below a threshold of approximately 2.3% but that an inertial Taylor-type rule has characterised its behaviour at higher rates of inflation.

Alongside these threshold models, a voluminous literature has grown around the notion of temporal change in the policy reaction function, perhaps driven by changes in the mandate of the central bank or in the nature of the macroeconomy. A recent example is provided by Raggi, Greco and Castelnuovo (2008), in which the authors estimate a Taylor rule with time-varying trend inflation where transitions between active and passive monetary policy regimes are governed by an unobserved underlying Markov chain. In order to estimate their model, the authors employ the popular Gibbs sampler in a Bayesian MCMC approach. Their results strongly suggest that the inflation target in the USA has been time-varying. Moreover, their state probabilities indicate, to a first approximation, that US monetary policy was passive between 1968 and 1975 and 1980-85 but that a modified Taylor principle was upheld elsewhere. Similarly, Petersen (2007) finds that the Fed followed a nonlinear Taylor rule under both Volcker and Greenspan but that monetary policy was linear in the pre-Volcker era. More specifically, he finds that, since 1985, the Fed has reacted more aggressively to inflation when it is at higher levels than when the price-level is growing slowly, with the transition from low to high inflation occurring between 3.3% and 3.8% in his smooth-transition framework. This leads him to conclude that nonlinearity is associated with enlightened policymaking.

Subject to the feasibility of an appropriate mapping between the time index and the covariates of the reaction function, such intertemporal regime-switching models can be related approximately to the asymmetric models discussed above. The general consensus to emerge from the regime-switching literature is that US monetary policy became increasingly anti-inflationary in the Volcker-Greenspan era. Moreover, a crude generalisation of the historical experience of US monetary policy may be that the Burns-Miller period was one of high inflation and a volatile output gap, the Greenspan-Bernanke era has been one of low inflation and greater economic stability (until recently at least) and the Volcker years account for the transition. Hence, it seems likely that results similar to those adduced by Raggi et al. could be achieved by a model in which state transitions are determined according to the behaviour of these core macroeconomic variables.

The papers surveyed above have dealt variously with what were termed reaction (long-run) and adjustment (short-run) asymmetries in the opening paragraph of this paper. However, it is possible that the response of the central bank to the inflation and output gaps may also depend

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1 By contrast, Surico (2007) identifies non-linearity with respect to the output gap in the pre-Volcker period only, and concludes that this form of asymmetry generated an average positive inflationary bias of 1.5% in the monetary policy of the time.

2 This consensus is not, however, absolute. Recently, Cukierman and Muscatelli (2008) have found that it was not inflation-avoidance but recession-avoidance that characterised the Greenspan years. Similarly, employing a novel approach to combining persistent and stationary series in a vector error correction model, Greenwood-Nimmo and Shin (2010) find that US monetary policy has been distinctly growth-oriented since the end of Volcker’s tenure and that there is little evidence that the Taylor principle has been observed post-Volcker. A similar conclusion is reached by Petersen (op. cit.), who concludes that the Taylor principle is not required for effective monetary policy if the reaction function is non-linear.
upon the level of the interest rate itself. The natural means by which to investigate such *loccational asymmetry* is by use of the quantile regression approach associated originally with Koenker and Bassett (1978) and subsequently with Koenker and Hallock (2001) and Koenker and Xiao (2006). As with the asymmetric models discussed above, subject to the feasibility of an appropriate mapping between the various quantiles of the dependent variable and the time index, the results of quantile regression models may be readily interpreted in terms of intertemporal state change. In the case of US monetary policy, such a mapping is easily achieved, as will be discussed shortly.

Symmetric quantile regression models have been widely used in a number of fields, notably the analysis of stock market returns (e.g. Barnes and Hughes, 2002) and in labour economics (e.g. Falaris, 2004; Martins and Pereira, 2004). However, at the time of writing, we are aware of only two papers that have applied quantile techniques to the analysis of monetary policy. Mizen, Kim and Thanaset (2009, hereafter MKT) consider the case of locational asymmetry at the Fed and the Bank of Japan in the context of an otherwise symmetrical forward-looking monetary policy rule. Their results indicate that the Taylor principle is upheld at every conditional quantile and that the degree of policy aggression, measured by the magnitude of the coefficient on inflation, is a monotonically increasing function of the conditional quantile of the interest rate.

More recently, Wolters (2009) has applied the quantile regression framework to the analysis of US monetary policy. His results suggest that the policy response to inflation increases over the conditional distribution of the Federal funds rate, while the reaction to output gap disequilibria decreases. In conjunction with the findings of MKT, these results clearly indicate that the widespread convention of modelling the policy rule at the conditional mean of the interest rate distribution may provide misleading results. However, neither Wolters nor MKT are able to convincingly address issues relating to reaction or adjustment asymmetries in their empirical frameworks. The development of a synthetic approach to the analysis of these three forms of nonlinearity is the focus of this paper. We will propose a simple means of combining the asymmetric ARDL approach originated by SYG with the quantile regression model, thereby achieving a tractable framework capable of modelling fundamentally asymmetric processes in a coherent and intuitively appealing manner.

### 3 The Asymmetric ARDL Model

SYG advance a simple technique for modelling both long- and short-run asymmetries in a coherent manner. The model is essentially an asymmetric extension of the linear ARDL approach to modelling long-run (cointegrating) levels relationships originated by Pesaran and Shin (1998) and Pesaran, Shin and Smith (2001, PSS). Consider the asymmetric cointegrating relationship:

$$y_t = \beta^+ x_t^+ + \beta^- x_t^- + u_t, \quad (3.1)$$

where $x_t$ is a $k \times 1$ vector of regressors decomposed as:

$$x_t = x_0 + x_t^+ + x_t^-,$$  \quad (3.2)

where $x_t^+$ and $x_t^-$ are partial sum processes of positive and negative changes in $x_t$ defined by:

$$x_t^+ = \sum_{j=1}^t \Delta x_j^+ = \sum_{j=1}^t \max (\Delta x_j, 0), \quad x_t^- = \sum_{j=1}^t \Delta x_j^- = \sum_{j=1}^t \min (\Delta x_j, 0), \quad (3.3)$$

and $\beta^+$ and $\beta^-$ are the associated asymmetric long-run parameters. The extension of (3.1) to the ARDL($p, q$) case is straightforward, yielding the following asymmetric error correction model:
\[
\Delta y_t = \rho y_{t-1} + \theta^+ x_{t-1}^+ + \theta^- x_{t-1}^- + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} (\pi_j^+ \Delta x_{t-j}^+ + \pi_j^- \Delta x_{t-j}^-) + \varepsilon_t.
\] (3.4)

We refer to (3.4) as the asymmetric or non-linear ARDL (NARDL) model. This approach has a number of advantages over the existing class of regime-switching models. Firstly, once the regressors, \(x_t\), are decomposed into \(x_t^+\) and \(x_t^-\), (3.4) can be estimated simply by standard OLS. Secondly, the null hypothesis of no long-run relationship between the levels of \(y_t\), \(x_t^+\) and \(x_t^-\) (i.e. \(\rho = \theta^+ = \theta^- = 0\)) can be easily tested using the bounds-testing procedure advanced by Pesaran, Shin and Smith (2001, PSS), which remains valid irrespective of whether the regressors are \(I(0)\), \(I(1)\) or mutually cointegrated. Thirdly, (3.4) nests the following two special cases: (i) long-run reaction symmetry where \(\theta^+ = \theta^- = \theta\), and (ii) short-run adjustment symmetry in which \(\pi_i^+ = \pi_i^-\) for all \(i = 0, \ldots, q\). Both types of restriction can be easily tested using standard Wald tests\(^3\). Only when these two restrictions are not rejected should the restricted linear ARDL\((p, q)\) model be entertained:

\[
\Delta y_t = \rho y_{t-1} + \theta x_{t-1} + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} \pi_j \Delta x_{t-j} + \varepsilon_t.
\] (3.5)

Finally, the asymmetric ARDL model, (3.4) can be used to derive the asymmetric cumulative dynamic multiplier effects of a unit change in \(x_t^+\) and \(x_t^-\) respectively on \(y_t\), defined by:

\[
m^+_h = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t^+}, \quad m^-_h = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t^-}, \quad h = 0, 1, 2, \ldots
\] (3.6)

Notice that, by construction, as \(h \to \infty\), \(m^+_h\) and \(m^-_h\) tend to approach the respective asymmetric long-run coefficients. At present, we evaluate the differential effects of positive and negative shocks to the explanatory variables under the assumption of a single known threshold value. Indeed, the construction of positive and negative partial sum processes relies on the imposition of a zero threshold. However, this assumption can be easily relaxed to accommodate the more general case of multiple unknown threshold decompositions (Greenwood-Nimmo, Shin and Van Treeck, 2009). Similarly, we currently work under the implicit assumption that positive and negative shocks to the explanatory variables occur with equal probability. In the current context this is a largely innocuous simplification as the mean values of \(\Delta \pi\) and \(\Delta y\) are relatively close to zero over our sample, implying that \(\Pr(\Delta x > 0) \approx \Pr(\Delta x < 0) \approx 0.5\). However, in the general case in which this condition is not satisfied, as with all regime-switching models, one must allow for the impact of the respective regime probabilities in the evaluation of the asymmetric dynamic multipliers.

The ability of the dynamic multipliers to illuminate the traverse between initial equilibrium, short-run disequilibrium following a shock, and a new long-run equilibrium makes them a powerful tool for the combined analysis of (short-run) adjustment asymmetry and (long-run) response asymmetry. This property is likely to prove particularly advantageous in the analysis of asymmetric central bank preferences.

\(^3\)SYG identify two different types of short-run symmetry restrictions: strong-form (pairwise) symmetry and weak-form (additive) symmetry. The former is a very strong restriction that is unlikely to be satisfied, particularly in the case of general-to-specific lag selection as this is likely to result in the inclusion of heterogeneous lags of the positive and negative partial sum process. While additive symmetry is a much weaker restriction, the power of the Wald test may be rather low in small samples, in which case the use of bootstrapped confidence intervals may be preferable.
3.1 The Quantile Extension of the NARDL Model

As MKT note, conventional regression techniques such as OLS, IV, or GMM evaluate the relationship between series at the mean of the conditional distribution of the dependent variable (p. 4). The implicit assumption is that the estimated relationship holds not only at the mean, but also in other parts of the conditional distribution of the dependent variable. In many cases, there is little reason to believe that this is an innocuous assumption. The relationship between series at the mean of the conditional distribution of the dependent variable may be written as:

\[ \Delta y_t = \rho(h)x_{t-1} + \theta^+(x_{t-1})x_{t-1} - \theta^-(x_{t-1})x_{t-1} + \sum_{j=1}^{p-1} \varphi(j)\Delta y_{t-j} + \sum_{j=0}^{q} (\pi^+(x_{t-j})\Delta x_{t-j} + \pi^-(x_{t-j})\Delta x_{t-j}) + \varepsilon_t \]

where \( \kappa \) is a given quantile index in \((0, 1)\), \( z_t \) is the vector of all regressors in the quantile model and \( \alpha(\kappa) \) is the vector obtained by collecting all the coefficients in the model. We impose the usual assumption that the conditional quantile model is correctly specified; that is,

\[ E(\psi_\kappa(\varepsilon_t)|z_t) = 0 \]

where \( \psi_\kappa(z) = \kappa - 1[z \leq 0] \). This assumption is equivalent to the following:

\[ \int_{-\infty}^{\psi^*_\kappa(z)} f_{\Delta y_t|z_t}(t|z_t)dt = \kappa \]

where the conditional density of \( f_{\Delta y_t|z_t}(t|z_t) \) is the density of \( \Delta y_t \) conditional on \( z_t \). Hence, it can be easily seen that this assumption implies that \( \psi^*_\kappa(\alpha(\kappa)) \) is the correct conditional quantile of \( \Delta y_t \) given \( z_t \) when the quantile index is given by \( \kappa \in (0, 1) \). Our objective is to analyse how \( z_t \) affects \( \Delta y_t \) over the range of the conditional distribution. This can achieved by estimating the conditional quantile for various values of \( \kappa \) over \((0, 1)\).

By admitting non-linearity of the form modelled by (3.4) into the conditional quantile function, we obtain the quantile-NARDL or NARDL-Q model. For a fixed value of \( \kappa \), the single-step quantile regression estimates of the model parameters are those values that minimise the following expression:

\[ \min_{\alpha(\kappa)} \sum_{t=1}^{T} \xi(z_t) \{ \Delta y_t - \psi^*_\kappa(\alpha(\kappa)) \} \]

(3.7)

where \( \xi(\kappa)(z) \) is the usual check function defined as \( \xi(\kappa)(z) = z(\kappa - 1[z \leq 0]) \) (c.f. Koenker and Hallock, 2001). The solution from this minimization, denoted \( \hat{\alpha}(\kappa) \), will be consistent and asymptotically normal under the assumption that the quantile specification is correct and subject to a number of mild regularity conditions. Finally, the dynamic multipliers associated with the \( h \) th conditional quantile of the dependent variable may be written as:

\[ m^+_{(\kappa)h} = \sum_{j=0}^{h} \frac{\partial y(\kappa)_{t+j}}{\partial x^+_{t}}, \quad m^-_{(\kappa)h} = \sum_{j=0}^{h} \frac{\partial y(\kappa)_{t+j}}{\partial x^-_{t}}, \quad h = 0, 1, 2... \]

(3.8)

Kim and Muller (2005, 2010) demonstrate that the single-step quantile estimation routine outlined above is biased when there exists non-zero contemporaneous correlation between the
explanatory variables and the residuals. This is likely to be particularly problematic for forward-looking models incorporating expectational terms, such as that developed by MKT. In this case, either the two-stage estimation procedure advanced by Kim and Muller or the inverse quantile regression technique of Chernozhukov and Hansen (2005) could be used to achieve reliable estimation. However, in this paper we do not consider forward-looking modelling and the ARDL model is known to correct perfectly for the endogeneity of $I(1)$ regressors (Pesaran and Shin, 1998). Therefore, we consider the single-step estimation routine sufficient for our present needs.

The NARDL-Q framework is able to explicitly model the following three types of asymmetry:

(i.) Reaction asymmetry - captured by the heterogeneous long-run parameters $\beta^+_{(\kappa)}$ and $\beta^-_{(\kappa)}$, this reflects the different long-run responses of the dependent variable to positive and negative changes in the explanatory variables.

(ii.) Adjustment asymmetry - captured by the differences between the estimated short-run parameters, $\pi^+_{(\kappa)j}$ and $\pi^-_{(\kappa)j}$ for $j = 0, ..., q$, this represents the differential impact effects of $x^+$ and $x^-$ on $y$ and the associated dynamic adjustment toward the respective long-run multipliers.

(iii.) Locational asymmetry - captured by the differences between the short- and long-run parameters estimated at various quantiles of the dependent variable, this relates to the changing response of the dependent variable to the explanatory variables at different values of $\kappa$.

In order to facilitate statistical discrimination between the various forms of asymmetry, we propose the following array of hypothesis tests:

(i.) $H_0: \beta^+_{(\kappa)} = \beta^-_{(\kappa)}$ vs. $H_1: \beta^+_{(\kappa)} \neq \beta^-_{(\kappa)}$.

(ii-a.) $H_0: \pi^+_{(\kappa)j} = \pi^-_{(\kappa)j}$ vs. $H_1: \pi^+_{(\kappa)j} \neq \pi^-_{(\kappa)j}$ for $j = 0, ..., q$.

(ii-b.) $H_0: \sum_j \pi^+_{(\kappa)j} = \sum_j \pi^-_{(\kappa)j}$ vs. $H_1: \sum_j \pi^+_{(\kappa)j} \neq \sum_j \pi^-_{(\kappa)j}$ for $j = 0, ..., q$.

It follows from SYG that the Wald statistics testing the null hypotheses of reaction symmetry and of pairwise and additive adjustment symmetry in the $\kappa$th conditional quantile will follow asymptotic $\chi^2$ distributions.

Further, we consider the following hypotheses:

(iii.) $H_0: \beta^+_{(h)} = \beta^+_{(k)}, \beta^-_{(h)} = \beta^-_{(k)}$ vs. $H_1: \beta^+_{(h)} \neq \beta^+_{(k)}, \beta^-_{(h)} \neq \beta^-_{(k)}$ for $h, k = \{0.05, \ldots, 0.95\}, h \neq k$. The test for long-run locational symmetry follows an asymptotic $\chi^2$ distribution.

(iv.) $H_0: \pi^+_{(h)j} = \pi^+_{(k)j}, \pi^-_{(h)j} = \pi^-_{(k)j}$ vs. $H_1: \pi^+_{(h)j} \neq \pi^+_{(k)j}, \theta^-_{(h)} \neq \theta^-_{(k)}$ for $h, k = \{0.05, \ldots, 0.95\}, h \neq k, j = 1, \ldots, q$. The test for short-run locational symmetry follows an asymptotic $\chi^2$ distribution.

While (i), (ii-a), (ii-b) focus on a given quantile index $\kappa$, (iii) and (iv) consider a range of quantile indices and test for the consistency of various model parameters over the range. For example, one can test whether the reaction asymmetries characterising the lower part of the conditional distribution (e.g. the lower 10% quantile) and those observed in the upper part (e.g. 90%) are common or heterogeneous. This can be expressed more formally as

$$H_0: \pi^+_{(0.1)} = \pi^+_{(0.9)} \text{ and } \pi^-_{(0.1)} = \pi^-_{(0.9)}.$$
For this test, it is necessary to estimate multiple quantiles simultaneously. The asymptotic normality of multiple quantiles is a well known result in the literature (e.g. Koenker and Bassett, 1978). For the above example, when $\kappa_i = 0.1$ and $\kappa_j = 0.9$, 

$$T^{1/2} \left( \frac{\hat{\pi}^+(\kappa_i)}{\hat{\pi}^+(\kappa_j)} - \frac{\hat{\pi}^-(\kappa_i)}{\hat{\pi}^-(\kappa_j)} \right) \Rightarrow N \left( \mathbf{0}_{2 \times 1}, \Omega \otimes Q \right)$$

where 

$$\Omega = [\omega_{ij}], \quad \omega_{ij} = \frac{\kappa_i (1 - \kappa_j)}{f(F^{-1}(\kappa_i))f(F^{-1}(\kappa_j))}$$

Hence, the Wald statistics testing the null hypothesis of locational symmetry between the $\kappa_i$th and $\kappa_j$th conditional quantiles will be asymptotically $\chi^2$ distributed. See Kim and Shin (2010) for further details.

## 4 Estimation Results

We estimate the asymmetric Taylor rule for the US between 1964q2 and 2008q2 using both the standard asymmetric ARDL framework derived by SYG (the NARDL-M model) and the NARDL-Q model described above\(^4\). The NARDL-M model evaluates the relationship between the variates at the conditional mean of the interest rate, as has become common practice in the empirical literature. By contrast, using the NARDL-Q model, we obtain estimates of the relationship at a range of quantiles across the entire conditional distribution. In this way we can investigate the response of the Federal Reserve to inflation and output at various levels of the interest rate, thereby shedding light on potential locational asymmetries and the nature of policymaking in the neighborhood of the zero nominal lower bound.

### 4.1 The NARDL-M Model

Table 1 presents the results of NARDL-M estimation of the asymmetric Taylor rule where $i_t$ denotes the short-term nominal interest rate, $\pi_t$ the rate of consumer price inflation, and $y_t$ the output gap\(^5\). The four columns of the table relate to the four general combinations of short- and long-run asymmetry identified by SYG. The PSS F-test identifies the existence of a long-run levels relationship at the 5% level in all cases. However, the long-run symmetry restrictions with respect to both inflation and the output gap cannot be rejected at the 5% confidence level regardless of the specification of the model dynamics. Furthermore, we find only weak support for additive adjustment asymmetry with respect to either inflationary shocks or output gap shocks. It is clear, however, that we observe pairwise asymmetric adjustment in columns 1 and 3 where short-run symmetry restrictions are not imposed during estimation\(^6\).

**TABLE 1 ABOUT HERE**

Figures 1 and 2 plot the cumulative dynamic multipliers associated with unit shocks to inflation and the output gap, respectively (or the associated positive and negative partial sum

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\(^4\)All data were retrieved from the IMF’s International Financial Statistics. Potential output was calculated using the Hodrick-Prescott filter with the smoothing parameter selected by the Ravn-Uhlig (2002) frequency rule.

\(^5\)In all cases, general-to-specific lag selection was performed starting from a maximum lag length of 4 using a sequential 5% rule as implemented by Gretl version 1.8.2cvs.

\(^6\)Note that the consideration of a broader range of models representing other feasible combinations of reaction and adjustment asymmetry on a variable-by-variable basis still provided little evidence of reaction asymmetry. The results of this analysis are available upon request.
processes). The dynamic response of the interest rate to the output gap is qualitatively similar under all specifications, suggesting that the response of the Federal Reserve to output gap disequilibrium is indeed linear at the conditional mean of the interest rate. By contrast, the dynamic multipliers obtained under the assumption of long-run symmetric responses to inflation gaps are quite different from those derived from the asymmetric case, indicating that the Fed has responded more aggressively to positive than to negative inflation gaps. The inability of the Wald test to reject the long-run symmetry restrictions in this case results from the presence of a non-negligible negative covariance. Hence, we are obliged to conclude that the NARDL-M model finds little evidence of asymmetry in the reaction function of the central bank. However, it remains to be seen whether this result may be safely generalised to the entire distribution of the interest rate.

FIGURES 1 & 2 ABOUT HERE

4.2 The NARDL-Q Model

We estimate the NARDL-Q model for \( \kappa = \{0.05, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95\} \) under the assumption of joint long- and short-run asymmetry where the lag structure is selected based on that presented in Table 1. Figures 3 - 5 plot the asymmetric cumulative dynamic multipliers with respect to inflation and output gap shocks at each conditional quantile of the interest rate. A general trend toward increasingly aggressive monetary policy as \( \kappa \) increases is evident in the figures.

FIGURES 3 - 5 ABOUT HERE

The finding that monetary policy has responded more robustly to inflation and the output gap when the interest rate is higher is comparable with the findings of MKT. However, in their paper, the authors find that the Fed’s policy aggression, measured by the magnitude of the inflation and output gap coefficients, is monotone increasing in \( \kappa \). Our results reveal a more complex relationship between \( \kappa \) and the coefficients of the reaction function. We find little difference between the reaction functions associated with \( 0.05 \leq \kappa \leq 0.4 \). In this region, our results provide little support for the operation of the Taylor principle in the USA and relatively little evidence of reaction or adjustment asymmetries in relation to either inflation or the output gap. For \( 0.4 < \kappa < 0.8 \), we observe a strong policy response to both positive and negative movements of the output gap and to positive inflationary pressures, with the Taylor principle upheld. By contrast, over this range of the interest rate, we note no significant response to disinflationary pressure over any horizon. Within this region, we find clear evidence of pronounced reaction asymmetry, suggesting that the Fed has systematically responded more strongly to positive than negative inflation gaps, and to negative than positive output gaps. Finally, for \( 0.8 \leq \kappa \leq 0.95 \), our results indicate very strong responses to both positive and negative inflation and output gaps and very pronounced asymmetry acting in the same direction as before.

Our conjectures based on these results are twofold. Firstly, it appears that US monetary policymakers act on the basis that economic agents do not respond to the absolute magnitude of a policy innovation but to its size in relation to the current level of the interest rate. Hence, while a 25 basis point rate cut may be considered substantial when the interest rate is initially at just 2%, the same intervention would be considered mild when the starting value of the interest rate is 10%. This is an intuitively reasonable finding when one considers the effect of the rate change on the nominal cost-of-capital. In layman’s terms, the former is equivalent to a 1/8 reduction in the cost-of-capital while the latter represents a reduction of just 1/40. When viewed in this
way, it is unsurprising that a larger interest rate change is required to achieve a given objective at higher levels of the interest rate.

Secondly, the observation that policy is conducted in a symmetrical fashion at low values of \( \kappa \) but asymmetrically at higher values suggests that policymakers become increasingly hawkish toward inflation and more concerned with negative output gaps at higher conditional quantiles of the interest rate. More specifically, while we observe relatively little variation between quantiles in terms of the impact effects of inflation and output gap shocks, profound asymmetries develop at longer horizons in the higher quantiles. These findings may be most easily explained by reference to the different historical episodes corresponding to the various quantiles of the interest rate, as shown in Figure 6. The shaded region in the figure denotes the 40-80% quantile range while the median is marked by the dashed line. For ease of interpretation, the transitions between chairmen are marked by vertical lines where the notation is self-explanatory\(^7\).

**FIGURE 6 ABOUT HERE**

Focusing initially on the lower quantiles where \( 0.05 \leq \kappa \leq 0.4 \), it is immediately apparent that the majority of observations populating this range relate to the so-called Great Moderation period and, to a lesser extent, to the relatively stable period of the late 60s and early 70s. In this range, as was noted above, we see little evidence of active anti-inflationary policies in conjunction with a modest policy response to the output gap. Moreover, we find little evidence of either reaction or adjustment asymmetries. For much of the 1960s, US inflationary pressures were restrained low and relatively stable and the persistence of inflationary shocks was limited by a range of factors, including globalisation and labour market reform. Certainly in the latter case, the Fed was able to act in an opportunistic fashion, often pursuing a growth-fostering mandate while allowing these external forces to restrain price-level inflation. Hence, we may conclude that for much of the Great Moderation period, policy acted in a relatively passive and linear fashion and was rather more similar to that enacted prior to the inflationary problems of the 1970s than is widely realised.

Moving on to the upper quantiles (\( 0.8 \leq \kappa \leq 0.95 \)), it is immediately apparent that the observations populating this range relate largely to the Volcker era and to a brief period of particularly high interest rates under Burns in the early to mid 1970s. It is well established that the principle concern of monetary policy in these difficult times was to contain burgeoning inflationary pressures and to attempt to avoid further growth of damaging inflationary spirals. It is, therefore, unsurprising that the estimated policy rule in this range lends excess weight to increasing inflation. Moreover, the highly persistent nature of inflationary shocks during this time is well-documented and this explains the zeal with which monetary policy strove to eliminate nascent inflationary pressures before they became entrenched.

The observation of a strongly asymmetric response to the output gap in the higher quantiles can be explained in relation to the notion that cash-flow constraints are more likely to be binding in the presence of high inflation rates and hence high nominal interest rates (Greenwald and Stiglitz, 2003, pp. 38-9). Although the firm’s investment is equally profitable in real terms, it faces a potential cash flow constraint wherever the lender is not committed to lend the difference between the return from the investment and the nominal debt-servicing obligations.\(^8\) Hence,

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\(^7\)Note that the Burns and Miller periods have been combined under the single heading “BUR” due to Miller’s very short tenure.

\(^8\)Suppose that a firm borrows $1,000 to buy an asset worth $2,000. Assume that the real interest rate and the rate of return are 5 and 10 per cent, respectively. With zero inflation, the nominal interest rate will be 5 per cent. After one year, the firm must pay $50 in interest, which can be easily covered with the cash flow earned from the asset ($200). Now, suppose that inflation increases to 20 per cent and hence the nominal interest rate is 25 per cent. In this case, the same cash flow of $200 is insufficient to pay the $250 owed in interest.
borrowing even at the same real interest rate becomes less attractive at high rates of inflation. The degree of uncertainty about future borrowing opportunities will typically be higher as the economy goes into a slump, in which case it follows that the Fed may wish to react more strongly to negative movements in the output gap at higher nominal interest rates.

Finally, consider the central quantiles \(0.4 < \kappa < 0.8\) represented by the shaded region of Figure 6. As noted above, within this range our results indicate that the Taylor principle has been upheld and that the central bank has reacted more strongly to positive than negative inflation gaps and also to negative than positive output gaps. The observations populating this quantile range are fairly widely dispersed, and relate largely to the latter years of the McChesney-Martin Fed, much of the Burns era and the period of transition between Volcker and Greenspan. Interestingly, we also find that the period of the late 90’s under Greenspan lies just within the lower end of this range despite the fact that it was during the Great Moderation. Within this central range, policymakers act relatively cautiously, resisting any tendency for accelerating inflation and stimulating economic growth actively in the face of recessionary pressures.

Given the close association between the nominal rate of interest and the inflation rate, we can generalise our findings as follows. In periods of low inflation, policy acts either passively or even opportunistically as the central bank takes advantage of forces outside their control that naturally tend to restrain inflation. Once inflationary pressures mount, policymakers start to act more conservatively to restrain price-level growth and support economic growth as the economy may be considered to be in a delicate state. Finally, once the rate of inflation becomes excessive, policymakers react very strongly and swiftly to bring it back under control and we observe very pronounced asymmetries reflecting this strong policy stance.

These observations are generally consistent with the results adduced in Greenwood-Nimmo, Shin and Van Treeck (2010, hereafter GST) and Greenwood-Nimmo and Shin (2010). GST employ the single equation ARDL-based error correction model developed by SYG in the analysis of asymmetric interest rate pass-through in the US and Germany. The results reveal a general pattern of short-run overshooting following a rate hike but only weak long-run pass-through. Moreover, the results indicate significant short-run under-reaction to expansionary policies followed by much stronger long-run pass-through. The overall pattern, therefore, is one in which rate hikes exert a stronger effect than rate cuts in the short-run but the opposite is true in the long-run. This combination of effects indicates that long-run inflation expectations have been contained during the period of the Great Moderation, with financial markets seemingly unwilling to consider the possibility that anti-inflationary interest rate hikes would be maintained over anything but a short horizon.

Greenwood-Nimmo and Shin estimate a novel VEC model combining the persistent and stationary variables entering Taylor’s rule on a rolling basis over precisely the same sample of data employed herein. The results of this exercise reveal a striking and clear pattern. Under Volcker, the Fed conducted strongly anti-inflationary monetary policy with relatively little concern for the output gap. Under Burns and Miller, the coefficients on inflation were considerably smaller and the Taylor Principle was often not upheld. Moreover, in this era, the results indicate a robust response to the output gap. Finally, in the Great Moderation period, the results provide only weak evidence of active anti-inflationary policies, with the Taylor principle frequently being neglected. The results presented by Cukierman and Muscatelli (2008) carry a similar implication. Although the methodology employed by Greenwood-Nimmo and Shin is strictly linear, the similarities with the results adduced herein are remarkable.

In the interest of clarity, we will now provide detailed results for the lower 10%, median, and upper 10% conditional quantiles. Each of these quantiles relates to a different type of Fed behaviour according to the simple three-way typology developed above and summarised in Figure 6. This exercise should, therefore, clarify the similarities and differences between the observed
behaviour of the Fed within qualitatively distinct ranges of the funds rate. Table 2 summarises the parameter estimates at the three selected quantiles. Note firstly that the pattern of significance of the regressors does not change materially between the selected quantiles, suggesting that our imposition of the lag structure derived from the NARDL-M model in estimation of the NARDL-Q model is generally appropriate.

TABLE 2 ABOUT HERE

Figure 7 plots the cumulative dynamic multipliers derived from the NARDL-Q model at the three selected quantiles. The patterns of dynamic adjustment to long-run equilibrium are quite striking and reveal the same pattern described by the three dimensional figures, but in somewhat more stark relief. The results displayed in panels (a) and (b) show that monetary policy has been largely symmetrical at the lower 10% quantile (which relates unambiguously to the low nominal interest rate era of the Great Moderation period), with at most very mild adjustment asymmetry confined to the short-run. Moreover, we note that the interest rate response to a positive inflation shock never exceeds unity, indicating that the Taylor principle is not upheld in this range.

FIGURE 7 ABOUT HERE

Focusing now on the median (which relates variously to the McChesney-Martin, Burns and Greenspan Feds), we observe pronounced long-run reaction asymmetry in relation to both inflation and output gap disequilibrium. More specifically, we find that the Taylor principle is satisfied in the case of positive inflationary pressure after a lag of approximately 10 quarters, reflecting inertial policymaking. By contrast, the interest rate response to disinflationary pressure remains negligible for the entire horizon. This is an interesting finding, which suggests that policymakers display pronounced inflation-aversion in this range which presumably reflects a belief that inflation has a tendency to rise more readily than it falls (Bunzel and Enders, 2005). That we find no evidence of rate cuts following reductions in the rate of inflation suggests that policymakers have been unwilling to risk a monetary easing in this range even when inflation seems to have subsided lest they precipitate a resurgence of underlying inflationary pressures. This certainly seems to have been the case during the late 1990s, when Greenspan maintained interest rates largely unchanged despite low and falling inflation.

In the case of the policy response to the output gap, we find that policymakers responded more strongly to negative than positive gaps at the median, reflecting the accepted wisdom that it takes a substantial rate cut to close a negative output gap but only a relative small rate rise to eliminate a positive gap. In this sense, it is often argued that attempting to correct a negative gap using monetary policy is akin to pushing on a string (this effect may also contribute to the observed short-run asymmetry in Figure 7(b)). On balance, therefore, it seems that policymakers have tended to act very cautiously in this range, suggesting that they are working on the premise that the economy is in a fragile state, prone to the influence of disequilibrating forces. The most obvious explanation of this behaviour is a fear of stagflation.

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Negative adjustment asymmetry in the case the output gap shock may result from the weak short-run pass-through of monetary policy to longer-term interest rates documented by GST. If the Fed is aware of this phenomenon, its interest rate response to a negative output gap shock should be proportionately stronger than in the case of a positive shock. By contrast, the positive asymmetry observed in the very short-run following an inflation shock probably reflects the Fed’s initial fear of accelerating inflation which gradually subsides once it becomes apparent that inflationary pressures have been contained in the longer-run due to a range of factors including globalisation and labour market reform. This can also explain why the longer-term response of the interest rate to a positive inflation shock has been rather muted.
Finally, consider Figures 7(e) and (f) relating to the upper 10% quantile. In this case, we observe very strong responses to both the output gap and inflation, and pronounced asymmetry in both cases. In fact, our results indicate that the long-run interest rate response to a unit positive inflation shock is approximately twice as large as the response to a unit negative shock. A similar pattern emerges in the case of output gap shocks. When we focus our attention acutely on just these three quantiles, a clear pattern emerges that is consistent with MKT’s argument that policy aggression is a monotonically increasing function of $\kappa$.

Overall, our results have two important implications. Firstly, as noted by MKT, policy does not become increasingly aggressive as the zero lower bound is approached; in fact, we observe the opposite. Secondly, by broadening our focus to the entire conditional distribution of the dependent variable, we are able to observe asymmetry where none was apparent when estimation was focused on the conditional mean. This suggests that the common practice of estimating at the conditional mean may obscure important underlying asymmetries.

5 Concluding Remarks

This paper identifies three fundamental forms of asymmetry that may characterise a dynamic economic process. Reaction asymmetry relates to the notion that the long-run response of the dependent variable to different types of shock to the same explanatory variable may differ. Adjustment asymmetry obtains when the path of dynamic adjustment of the dependent variable differs according to the nature of a shock to a given explanatory variable. Finally, locational asymmetry occurs when the response of the dependent variable to a given shock depends upon the conditional quantile of the dependent variable.

It follows that an easily implemented approach to modelling these three forms of asymmetry simultaneously and in a coherent fashion could be put to an abundance of uses. To this end, we develop a quantile regression extension of the asymmetric ARDL framework advanced by Shin, Yu and Greenwood-Nimmo (2009). More exactly, we specify a nonlinear conditional quantile function using the asymmetric ARDL functional form. Based on this structure, we can compute asymmetric cumulative dynamic multipliers with which to analyse response and adjustment asymmetries at the conditional quantile of interest. Moreover, we propose an array of hypothesis tests relating to each form of asymmetry in order to place the resulting modelling framework on a firm statistical footing.

Applying this technique to the analysis of US monetary policy, we find that the Federal Reserve responds linearly to both output and inflation in the lower quantiles of the interest rate. Moreover, in this range, the Fed does not adhere to the Taylor principle, indicating that monetary policy would not generally be considered stabilising in this region. We attribute this seemingly strange behaviour to the fact that the lowest quantiles of the interest rate in our sample relate mainly to the Greenspan-Bernanke period and the so-called Great Moderation. We argue that our results simply reflect the low long-run inflationary pressures that dominated at this time due to the effects of globalisation, financial innovation and far-reaching labour market reforms that served to contain wage inflation. Moreover, we note that although we find little evidence that the Taylor principle was upheld at this time, this finding is not inconsistent with good policy management when one considers that small interest rate hikes by the Fed sufficed to raise long-term interest rates in the short-run and alleviate inflationary pressures in the long-run during this period (c.f. GST). This interpretation is strengthened by the fact that the estimated impact effect of an inflationary shock is roughly homogeneous at all quantiles.

Between the fortieth and eightieth quantiles, we find that the Taylor principle is upheld in the case of positive inflationary shocks but not in response to disinflationary shocks. Meanwhile, we note significant responses to both positive and negative output gap shocks, with a marked nega-
tive asymmetry. Hence, we conclude that the Fed has acted as an inflation hawk in this region while also displaying a marked tendency toward growth-fostering policies. This combination of policies is broadly consistent with the opportunistic approach to monetary policy documented by Orphanides and Wilcox (2002). Furthermore, it is interesting to note that many of the observations populating these quantiles relate to the Burns era, which saw the build-up of persistent inflationary pressures against a backdrop of weak real activity. Many commentators have argued that this economic malaise was caused (or at least exacerbated) by the failure of the monetary policy of the time to adhere to the Taylor principle. Our results do not support this hypothesis.

Finally, for the uppermost quantiles relating mainly to the Volcker era, we find evidence of very aggressive policy responses to positive and negative changes in the rate of inflation and in the output gap in the context of profound reaction asymmetry. Overall, therefore, our results support MKTs finding that the degree of policy aggression is an increasing function of $\kappa$, although we must challenge Wolters’ finding that the policy response to the output gap decreases over the conditional distribution. It is likely that the difference between our respective findings reflects SYG’s argument that the failure to account for underlying asymmetries may engender profound biases in estimated linear models.

Our results have a number of important implications for the conduct of monetary policy. Firstly, the finding that policy is relatively passive in the lowest quantiles would typically be taken to suggest that the Fed has failed to pursue an optimal policy in this range. While the downside movement of the interest rate may be constrained to some degree by the proximity of the zero lower bound in this case (although US interest rates have never fallen much below 1% in our sample), a weak response to positive inflation or output gap shocks is also intuitively reasonable. At very low nominal interest rates, inflation is typically low (below the de facto target level) and the output gap negative, in which case higher inflation and an increasing output gap can be tolerated without running the risk of accelerating inflationary pressures or overheating the economy. Moreover, where inflationary pressures are weak and their persistence low, good monetary policy should act opportunistically to promote economic growth.

Another more general implication of our results is that the actions of the Fed become increasingly asymmetric as the interest rate increases. When one considers that higher nominal interest rates have been associated with high and persistent inflation, it follows that policymakers must greet further inflationary pressures with decisive intervention but that they will be reluctant to cut rates aggressively in response to falling inflation lest they re-start the inflationary spiral. Furthermore, the presence of cash-flow constraints that become increasingly binding at high nominal interest rates (typically associated with high inflation rates) may explain why the Fed has tended to react more strongly to negative output gap shocks at higher nominal interest rates. This is particularly plausible given that the higher quantiles of the interest rate in our sample relate to periods of stagflation, in which the economic outlook was at best fragile, if not bleak.

Finally, we will close with a general observation regarding the combined modelling of various asymmetries. The failure of the NARDL-M model to reject the null hypotheses of reaction and adjustment symmetries leads us to believe that the common practice of confining one’s attention to the mean of the conditional distribution of the dependent variable may obscure important underlying effects. Hence, it follows that combination of the NARDL technique with quantile estimation (the NARDL-Q model) may provide profound insights into a range of, as yet, poorly understood economic phenomena.
References


### (A) Estimation Results

<table>
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<th>Regressor</th>
<th>LR &amp; SR asym</th>
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<th>LR sym &amp; SR asym</th>
<th>LR &amp; SR sym</th>
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### (B) Diagnostic and Inferential Test Statistics

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<th>$\chi^2_{HET}$</th>
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**Note:** $\chi^2_{SC}$ and $\chi^2_{HET}$ denote the Breusch-Godfrey LM test for serial correlation and the White LM test for heteroscedasticity. $W_{LR,\pi}$ refers to the Wald test of the restriction $\beta_{+\pi} = \beta_{-\pi}$ while $W_{LR,y}$ refers to the Wald test of $\beta_{y^+} = \beta_{y^-}$. By analogy, $W_{SR,\pi}$ and $W_{SR,y}$ are the Wald tests for additive adjustment asymmetry. The relevant 5% critical value of the $F_{PWS}$ test is 4.01 for $k = 4$ and 4.85 for $k = 2$.

Table 1: Estimation Results for the NARDL-M Model
<table>
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<th>Regressor</th>
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<th>Median Quantile</th>
<th>Upper 10% Quantile</th>
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Table 2: Estimation Results for the NARDL-Q Model
Figure 1: Dynamic Multipliers for the NARDL-M Model: Inflation Shock

Figure 2: Dynamic Multipliers for the NARDL-M Model: Output Gap Shock
Figure 3: NARDL-Q Dynamic Multipliers of Positive Inflation and Output Gap Shocks
Figure 4: NARDL-Q Dynamic Multipliers of Negative Inflation and Output Gap Shocks
(a) Asymmetry: inflation shock

(b) Asymmetry: output gap shock

(c) Cross-sectional profiles of inflation asymmetry

(d) Cross-sectional profiles of output gap asymmetry

Figure 5: NARDL-Q Dynamic Multipliers - Asymmetry (i.e. the difference between Figures 3 and 4)
Figure 6: Unconditional Quantiles of the Federal Funds Rate
Figure 7: NARDL-Q Dynamic Multipliers: $\kappa = 0.1, 0.5, 0.9$. 