The Asymmetric ARDL Model with Multiple Unknown Threshold Decompositions: An Application to the Phillips Curve in Canada*

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Abstract

Through its many incarnations, the Phillips curve has remained a fundamental cornerstone of the macroeconomic debate, a position that is assured by its role in informing the design and conduct of stabilisation policies. However, the evolution of the Phillips curve is an ongoing process and a number of recent studies have suggested that it may exhibit various non-linearities. This observation has profound implications for monetary policy as nonlinearity in the underlying Phillips curve will result in nonlinear monetary policy even when policymakers' preferences are strictly quadratic. We estimate a threshold-ARDL model of the Canadian Phillips curve and find overwhelming support for asymmetry in relation to the rate of change of unemployment. Our results suggest that the output-unemployment tradeoff is steep when unemployment is changing rapidly but insignificant when unemployment is changing slowly. Hence, we conclude that there is substantial scope for the Bank of Canada to pursue opportunistic expansionary policies when the unemployment rate is in the relatively stable regime provided that it does not propel the economy into either of the outer regimes insodoing.

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1 Introduction

Since its inception, the Phillips curve has remained at the centre of the macroeconomic debate. The simple proposition that there exists a somewhat regular and perhaps predictable tradeoff between inflation and unemployment has profound implications for the conduct of macroeconomic stabilisation policies. At a superficial level, the simple linear form of Phillips curve suggests that policymakers can directly trade lower unemployment against higher inflation and *vice-versa*, at least in the short-run. In reality, however, experience suggests that the exploitation of any inverse relationship between inflation and unemployment may be complicated by a raft of factors including the necessity to accurately estimate the NAIRU, the difficulties posed by lags in the transmission mechanism from aggregate activity to prices and the sectoral and regional heterogeneity characterising many modern economies, to name but a few.

A growing body of evidence has challenged the notion that the Phillips curve is linear, arguing variously that it may exhibit a wide range of forms including convexity, concavity and piecewise linearity. This observation has far-reaching ramifications for monetary policy as nonlinearity in the underlying Phillips curve will result in nonlinear monetary policy even when policymakers' preferences are strictly quadratic. Moreover, the uncertainty surrounding the form of nonlinearity poses a significant challenge to policymakers as the optimal policy stance under convexity, for example, will be entirely inappropriate under concavity.

In this paper, we argue that the problems do not end here. It seems plausible that the relationship between unemployment and inflation may depend crucially on a wide range of factors including the degree of macroeconomic uncertainty, particularly in the labour market. We investigate this proposition using a threshold-ARDL model of the Canadian Phillips curve and find overwhelming support for asymmetry in relation to the rate of change of unemployment used as a proxy for uncertainty. In particular, our results suggest that the output-unemployment tradeoff is steep when unemployment is changing rapidly in either direction but that there is no significant tradeoff when unemployment is changing slowly.

We explain these observations in terms of both demand-side and supply-side effects. On the demand-side, it is widely acknowledged that households and firms tend to adopt more conservative spending plans as uncertainty mounts and that demand tends to expand in the boom phase as a more optimistic and even euphoric mood takes hold. Assuming that the rate of change of unemployment conveys useful information about the economic outlook, it follows that the expectations of rational economic agents should respond strongly to these signals, thereby contributing to a demand-pull inflationary process.

On the supply side, the rate of change of unemployment has powerful implications for the balance of power between employees and employers. When unemployment is rising rapidly, employers find themselves in a relatively strong position to restrain wage growth and are, therefore, able to maintain a low rate of output price inflation. By contrast, when the unemployment rate is falling rapidly, the balance of power shifts in favour of employees, resulting in more generous pay settlements and fueling cost-push inflationary pressures.

Finally, we note that the central regime reflects an intermediate position in which the mood of households and firms is neither pessimistic nor euphoric and the balance of power favours neither employers nor employees disproportionately. In such an environment, the economy is in a fairly stable condition an the labour market is able to accommodate small adjustments with little inflationary response. However, our estimates suggest that this central regime is rather narrow in Canada, operating when the absolute value of the annual change in unemployment is less than 0.12 percentage points. Hence, we conclude that there is substantial scope for the Bank of Canada to pursue opportunistic expansionary policies when the unemployment rate is within this relatively stable regime provided that it does not propel the economy into either of the outer regimes insodoing.

2 A Nonlinear Phillips Curve

The negative relationship between unemployment and inflation embodied in the Phillips curve is among the most well-documented phenomena in modern macroeconomics. From it's inception by A.W. Phillips in 1958 as a relationship between wage-level inflation and unemployment, through the Samuelson-Solow (1960) synthesis in which it was recast in terms of price-level inflation, from the proposal of the expectations-augmented (short-run) form by Friedman and Phelps (Phelps, 1967; Friedman, 1968) to the development of its modern forward-looking New Keynesian specification (Clarida, Galí and Gertler, 1999), the Phillips curve has always remained at the heart of the macroeconomic debate. The reason for this is simple: the tradeoff between inflation and unemployment (or output) fundamentally informs the design and conduct of macroeconomic stabilisation policies. Governments have risen and fallen on the basis of their manipulation of this apparent tradeoff and it remains the ever-present concern of the central banker.

Researchers and practitioners alike are increasingly considering the possibility that the Phillips curve may exhibit various forms of nonlinearity. Initially, a weak consensus developed around the notion of a convex Phillips curve. However, in his influential contribution to the 1997 *Journal of Economic Perspectives* symposium on the natural rate hypothesis, Joe Stiglitz raised the possibility that the Phillips curve may be kinked at the NAIRU in such a way as to introduce a piecewise linear concavity to the inflation-unemployment tradeoff. Meanwhile, yet another group adhered to the notion of linearity (*e.g.* Gordon, 1997). Given the aforementioned importance of the Phillips curve, it is clear that this debate has a strong bearing on the conduct of monetary policy. The implication of convexity is that the employment loss associated with a given disinflation is likely to outweigh the employment gain from an equivalent inflationary episode. The reverse is true if the Phillips curve is concave, while the tradeoff is constant in the case of linearity. Hence, a risk averse policymaker will act conservatively if they believe in convexity and more experimentally if they believe in concavity (Stiglitz, p. 10). It is, therefore, not surprising that considerable research effort has been devoted to discriminating between these competing forms of nonlinearity by empirical means.

Empirical evidence in support of a convex Phillips curve has been provided by Debelle and Laxton (1997), Laxton, Rose and Tambakis (1999) and, in the case of the Eurozone, by Dolado, Maria-Dolores and Naveira (2005). Debelle and Laxton argue that many studies that have rejected convexity have employed measures of the NAIRU that are fundamentally incompatible with a Phillips curve of this shape. The authors address this issue by generating model consistent estimates of the NAIRU simultaneously with the model parameters using Kalman filtering. Applying this technique to both a convex and linear form for the UK, USA and Canada, they find that the nonlinear model outperforms its counterpart in all cases. Laxton *et al.* continue in this vein, demonstrating by a Monte Carlo experiment that traditional econometric methods may have low power to identify modest convexity of the Phillips curve. Chief among the failings of the established methods identified by the authors are the use of backward-looking expectations and imprecision in the measurement of excess demand. In the European case, Dolado *et al.* estimate a simple quadratic Phillips curve for Germany, France, Spain, the Euro Area and the USA and find that the quadratic term is positive and statistically significant in the European countries but not the USA. These results imply an underlying convexity in the European countries that the authors attribute to labour market rigidities unique to this group. Finally, in a very thorough analysis of the Canadian data, Dupasquier and Ricketts (1998) find modest empirical support for various types of non-linearity, although they are unable to distinguish between the competing theories with any certainty[1]. In general, their results are consistent with the capacity constraints

[1] In his discussion of their paper, Nicholas Rowe (1997) makes the obvious but often-overlooked point that nonlinearity in the unemployment-inflation tradeoff does not necessarily imply nonlinearity in the output-inflation

model (implying a convex Phillips curve), the misperceptions model and the costly adjustment model.

In addition to the estimates of the Council of Economic Advisers cited in Stiglitz (1997), further evidence favouring the concave form has been adduced by Akerlof, Dickens and Perry (1996), Eisner (1996, 1997) and Coen, Eisner, Tepper Marlin and Shah (1999). These papers typically employ simple linear regression techniques on data decomposed on the basis of external estimates of the (time-varying) NAIRU. The nature of the unemployment-inflation tradeoff is then evaluated for two distinct regimes, one in which unemployment exceeds the (time-varying) NAIRU and the other where it lies below it. The general finding is that low levels of unemployment have not typically been associated with accelerating inflation. Indeed, Eisner (1997) finds a positive relationship between unemployment and inflation for some periods. Coen *et al.* contend that much of the evidence suggesting that unemployment rates below the NAIRU lead to accelerating inflation has been derived on the basis of extrapolation from linear models fitted to data that does not typically contain much information about such low unemployment regimes. Rather colourfully, they liken economists' concerns about accelerating inflation to ancient seafarers' fears of falling over the edge of the Earth if they ventured too far; fears that were clearly based on spurious extrapolation due to the lack of experience or evidence (p. 52).

Given this mixed empirical evidence, nonlinearity remains a controversial topic. A number of papers have argued that the observation of nonlinearity derives from a failure to account for important underlying effects. For example, Musso, Stracca and van Dijk (2007) identify a mean shift in European inflation and argue that while the Phillips curve has become shallower in recent years it is not asymmetric. An appealing alternative means of reconciling the contradictory empirical results has been proposed by Filardo (1998) and Freedman, Harcourt and Kriesler (2004). The authors argue that the Phillips curve may be both convex and concave at different levels of economic slack. This leads Filardo to propose that there exists a concave zone when the output gap is negative (high unemployment) and a convex zone when it is positive (low unemployment). More generally, this suggests that there may be an intermediate range in which the inflation-unemployment tradeoff is not particularly acute. It follows, therefore, that the optimal policy stance will be contingent on the stage of the business cycle.

Filardo's own estimates suggest that the convex zone exhibits a steeper gradient than the concave zone, suggesting that policymakers must be aware of incipient inflationary pressures in economic booms as these may be a bigger threat than deflationary pressures in a slump. Further

relationship in the case of a diminishing marginal product of labour.

empirical support for complex nonlinear Phillips curves has been provided by Eliasson (2001), Baghli, Cahn and Fraisse (2007) and Huh, Lee and Lee (2008). Eliasson estimates a smooth transition model for Australia, Sweden and the USA and finds evidence of nonlinearity in all cases but the latter. By contrast, Huh, Lee and Lee (2008) develop an LSTAR model and find evidence in support of nonlinearity in the US Phillips curve. Baghli, Cahn and Fraisse (2007) find robust evidence of nonlinearity in Europe both at an aggregate level and at the national level for France, Germany and Italy. Employing the non-parametric Nadaraya-Watson kernel estimator, the authors find overwhelming support for a sigmoid (concave-convex) Phillips curve. A particularly appealing feature of these models is that they do not require the econometrician to impose a known form of asymmetry *a priori*, but rather admit a range of possible nonlinearities.

While the concave-convex Phillips curve seems capable of reconciling some of the conflicting empirical evidence surveyed above, it remains a relatively simple treatment of non-linearity in the sense that it focuses on the levels of inflation and unemployment. However, given that inflation is influenced strongly by the pricing decisions of firms and the outcome of the wage-bargaining process, it follows that the degree of uncertainty in the job market may also play a significant role in determining the nature of the inflation-unemployment tradeoff. One means of capturing such an effect is by considering nonlinearity in relation to the rate of change of unemployment where the intuition is that uncertainty increases in proportion to the rate at which the unemployment rate is changing. Depending upon the dynamic behaviour of the unemployment rate, this form of asymmetry is not necessarily inconsistent with any of those already observed in the empirical literature. Equally, it does not lend direct support to any of them either.

Our proposed form of asymmetry can be rationalised in at least two ways. Firstly, and perhaps most obviously, one can extend the menu cost literature associated with Mankiw (1985) to the case of wages. In this environment, one may argue that there is a cost associated with re-negotiating wage settlements in response to labour market conditions so workers and firms will only act if these conditions change sufficiently to make such action worthwhile. Chief among these adjustment costs may be established social norms and the practice of long-term contracting that is prevalent in the labour market. Secondly, one may consider a simple behavioural explanation whereby firms and workers either do not perceive small changes in the rate of unemployment or consider them insignificant and perhaps temporary. In this relatively stable regime, neither firms nor workers have excessive bargaining power and so the wage- and price-levels may be expected to evolve gradually. However, outside this stable regime, when unemployment is increasing (decreasing) rapidly, it follows that employers (employees) enjoy the balance of power and that wage growth will be restrained (rapid).

This form of asymmetry provides substantial scope for opportunistic policymaking in the sense of Orphanides and Wilcox (2002) in the stable central regime. Subject to an accurate assessment of the thresholds defining this regime, policymakers would enjoy the latitude to reduce interest rates in order to foster economic growth without fear of the inflationary consequences. However, policymakers must remain alert to the signs that they have exceeded the thresholds defining the tranquil regime or else they may introduce significant inflationary/deflationary pressures into the economy.

3 Asymmetric ARDL with Multiple Threshold Decompositions

The asymmetric ARDL model advanced by Shin, Yu and Greenwood-Nimmo (2011, hereafter SYG) represents a natural means of modelling a Phillips curve that is nonlinear in relation to the rate of change of unemployment. The asymmetric ARDL model combines a nonlinear long-run (cointegrating) relationship with nonlinear error correction by use of carefully constructed partial sum decompositions. Consider the asymmetric long-run relationship:

$$y_t = \boldsymbol{\beta}^{+\prime}\boldsymbol{x}_t^+ + \boldsymbol{\beta}^{\prime -}\boldsymbol{x}_t^- + u_t, \tag{3.1}$$

where $\boldsymbol{x}_t$ is a $k \times 1$ vector of regressors decomposed as

$$\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{x}_t^+ + \boldsymbol{x}_t^-, \tag{3.2}$$

where $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$ are partial sum processes of positive and negative changes in $\boldsymbol{x}_t$ defined by

$$\boldsymbol{x}_t^+ = \sum_{j=1}^{t} \Delta\boldsymbol{x}_j^+ = \sum_{j=1}^{t} \max\left(\Delta\boldsymbol{x}_j, 0\right), \; \boldsymbol{x}_t^- = \sum_{j=1}^{t} \boldsymbol{\Delta} x_j^- = \sum_{j=1}^{t} \min\left(\Delta\boldsymbol{x}_j, 0\right), \tag{3.3}$$

and $\boldsymbol{\beta}^+$, $\boldsymbol{\beta}^-$ are the associated asymmetric long-run parameters. SYG demonstrate that the model can be written in error-correction form as follows:

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta}^+\boldsymbol{x}_{t-1}^+ + \boldsymbol{\theta}^-\boldsymbol{x}_{t-1}^- + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} \left(\boldsymbol{\pi}_j^+ \Delta\boldsymbol{x}_{t-j}^+ + \boldsymbol{\pi}_j^- \Delta\boldsymbol{x}_{t-j}^-\right) + \varepsilon_t. \tag{3.4}$$

where both the long-run equilibrium relationship and the dynamic adjustment process are allowed to vary between the regimes defined by the partial sums. In this framework, the nonstandard bounds-based F-test of the null hypothesis $\rho = \boldsymbol{\theta}^{+} = \boldsymbol{\theta}^{-} = 0$ can be applied to test for the existence of an asymmetric long-run levels relationship (Pesaran, Shin and Smith, 2001). This approach is valid irrespective of whether the regressors are $I(0)$, $I(1)$ or mutually cointegrated. Similarly, (3.4) nests the following three special cases: (i) long-run symmetry where $\boldsymbol{\theta}^{+} = \boldsymbol{\theta}^{-} = \boldsymbol{\theta}$; (ii) short-run symmetry in which $\sum_{i}^{q} \boldsymbol{\pi}_{i}^{+} = \sum_{i}^{q} \boldsymbol{\pi}_{i}^{-}$; and (iii.) the combination of long- and short-run symmetry in which case the model collapses to the standard symmetric ARDL model advanced by Pesaran and Shin (1998)[2] Both types of restriction can be easily tested using standard Wald tests. Finally, the traverse between short-run disequilibrium and the new long-run steady state of the system can be described as follows by the asymmetric cumulative dynamic multipliers:

$$\boldsymbol{m}_{h}^{+} = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial \boldsymbol{x}_{t}^{+}}, \ \boldsymbol{m}_{h}^{-} = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial \boldsymbol{x}_{t}^{-}}, \ h = 0, 1, 2... \tag{3.5}$$

where $\boldsymbol{m}_{h}^{+}$ and $\boldsymbol{m}_{h}^{-}$ tend toward the respective asymmetric long-run coefficients $\boldsymbol{\beta}^{+} = \boldsymbol{\theta}^{+}/-\rho$ and $\boldsymbol{\beta}^{-} = \boldsymbol{\theta}^{-}/-\rho$, respectively, as $h \to \infty$. The ability of the dynamic multipliers to illuminate the traverse between steady states is likely to prove particularly useful in our analysis of the Phillips curve, providing insights into the dynamics of the inflation-unemployment tradeoff.

To this point, we have assumed that $\boldsymbol{x}_t$ is decomposed into $\boldsymbol{x}_t^{+}$ and $\boldsymbol{x}_t^{-}$ about a zero threshold value delineating the positive and negative changes of the growth rate of $\boldsymbol{x}_t$. This simple approach has an intuitive appeal and provides estimation results that may be easily interpreted, particularly in relation to expansionary or contractionary periods of the business cycle and the arrival of good and bad financial news, for example. However, in the case where the growth rates of time series of interest are predominantly positive (negative), this may result in a situation where the number of effective observations in the negative (positive) regime is insufficient for the OLS estimator to be well determined (*i.e.* the use of a zero threshold may introduce a finite sample problem in one regime in this case).

A more general approach to the construction of the partial sum processes employing a non-zero threshold, d, may help to avoid this problem. In this case, x_t is decomposed as:

$$\Delta x_t^{+} = \max\left(\Delta x_t, d\right) \text{ and } \Delta x_t^{-} = \min\left(\Delta x_t, d\right), \tag{3.6}$$

where x_t follows a random walk process with a drift, g:

[2]SYG refer to this form of short-run symmetry as weak-form or additive symmetry. An alternative stronger-form of short-run symmetry arises in the case of pairwise restrictions of the form $\boldsymbol{\pi}_{i}^{+} = \boldsymbol{\pi}_{i}^{-}$ for all $i = 0, ..., q$.

$$\Delta x_t = g + v_t. \tag{3.7}$$

When the threshold parameter, d, is known, OLS estimation can be carried out as usual and standard inference remains valid. An obvious candidate value of d is the mean value of the first differences of the series of interest. Using this approach, the number of effective observations in each regime should be approximately equal in most cases. When d is unknown, it can be consistently estimated using a grid search algorithm over the transition variable:

$$\hat{d} = \underset{d \in D}{\operatorname{argmin}} \; Q\left(d\right), \tag{3.8}$$

where $Q\left(d\right)$ is the sum of squared residuals of the OLS regression associated with a particular value of $d \in D$, with D being the grid set consisting of the partial support of the transition variable after 'trimming' extreme observations (the established practice is to trim at the 15th and 85th percentiles). Following Hansen (2000), it is possible to construct the confidence interval for $\hat{d}$ by forming the non-rejection region using the LR statistic for the null hypothesis, $H_0 : d = d_0$. The long-run symmetry restrictions, $\boldsymbol{\theta}^+ = \boldsymbol{\theta}^- = \boldsymbol{\theta}$ and the short-run symmetry restrictions, $\boldsymbol{\pi}_i^+ = \boldsymbol{\pi}_i^-$ can still be tested using the Wald statistic, although its asymptotic null distribution will be nonstandard due to the well-known problem that the nuisance (threshold) parameter is unidentified under the null (the Davies Problem - see Davies, 1987). Hence, its asymptotic p-value must be evaluated by either bootstrapping or by the sub-sampling approach discussed by Shin (2008).

In the most general case, an asymmetric long-run relationship may be defined between y_t and any number of partial sum processes of $\boldsymbol{x}_t$ as follows:

$$y_t = \boldsymbol{\beta}_1' \boldsymbol{x}_t^{(1)} + \boldsymbol{\beta}_2' \boldsymbol{x}_t^{(2)} + ... + \boldsymbol{\beta}_S' \boldsymbol{x}_t^{(S)} + u_t, \tag{3.9}$$

where $\boldsymbol{x}_t$ is a $k \times 1$ vector of regressors decomposed into S component series based on $S-1$ thresholds (which may be either known or unknown) as follows:

$$\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{x}_t^{(1)} + \boldsymbol{x}_t^{(2)} + ... + \boldsymbol{x}_t^{(S)}, \tag{3.10}$$

$$\boldsymbol{x}_t^{(1)} = \sum_{j=1}^{t} \Delta \boldsymbol{x}_j 1_{\{\Delta \boldsymbol{x}_j < d_1\}},\ \boldsymbol{x}_t^{(s)} = \sum_{j=1}^{t} \Delta \boldsymbol{x}_j 1_{\{d_{s-1} \leq \Delta \boldsymbol{x}_j \leq d_s\}},\ s = 2, ..., S-1,\ \boldsymbol{x}_t^{(S)} = \sum_{j=1}^{t} \Delta \boldsymbol{x}_j 1_{\{d_{S-1} < \Delta \boldsymbol{x}_j\}}, \tag{3.11}$$

where $1_{\{A\}}$ is an indicator function taking the value of unity if the condition A is satisfied and 0 otherwise.

In the case of multiple unknown thresholds, consistent estimation can be achieved through searching over an $S-1$ dimensional hypercube defined over the partial support of the distribution of the variable of interest. However, it is typically necessary to reject certain combinations of grid coordinates in order to ensure that estimation remains feasible for all S regimes. In particular, it is necessary to ensure that the number of effective observations in each regime is sufficient for estimation. Moreover, a range of logical restrictions may be required to ensure that, for example, $d_1 < d_2 < ... < d_{S-1}$, such that the associated partial sum processes maintain an economically meaningful interpretation.

The first-best method of estimating multiple unknown thresholds is to search for them simultaneously and select whichever set is associated with a global inferiorum of the sum of squared residuals of OLS estimation of the asymmetric ARDL model. An alternative approach, proposed by Hansen (1999) for use in computationally demanding cases, is to search sequentially over the grid set, estimating one threshold and then fixing its value to estimate another, before finally re-estimating the first. The methods are asymptotically equivalent. Of course, as with the case of a single unknown threshold, the distribution of the common inferential statistics will depend on the threshold parameters and so reliable inference can only be achieved by use of re-sampling techniques.

3.1 Computational Details

The three models considered here are the linear symmetric ARDL(p, q) model, the single-threshold ARDL(p, q, q) model and the double-threshold ARDL(p, q, q, q) model. These may be written as follows:

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta} \boldsymbol{x}_{t-1} + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} \boldsymbol{\pi}_j \Delta \boldsymbol{x}_{t-j} + \varepsilon_t \tag{3.12}$$

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta}^+ \boldsymbol{x}_{t-1}^+ + \boldsymbol{\theta}^- \boldsymbol{x}_{t-1}^- + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} \left(\boldsymbol{\pi}_j^+ \Delta \boldsymbol{x}_{t-j}^+ + \boldsymbol{\pi}_j^- \Delta \boldsymbol{x}_{t-j}^- \right) + \varepsilon_t \tag{3.13}$$

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta}^A \boldsymbol{x}_{t-1}^A + \boldsymbol{\theta}^B \boldsymbol{x}_{t-1}^B + \boldsymbol{\theta}^C \boldsymbol{x}_{t-1}^C + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \sum_{j=0}^{q} \left(\boldsymbol{\pi}_j^A \Delta \boldsymbol{x}_{t-j}^A + \boldsymbol{\pi}_j^B \Delta \boldsymbol{x}_{t-j}^B + \boldsymbol{\pi}_j^C \Delta \boldsymbol{x}_{t-j}^C \right) + \varepsilon_t \tag{3.14}$$

where the partial sum processes are defined in relation to the single threshold d in (3.13) and the two thresholds d_1 and d_2 in (3.14). We will henceforth refer to the ARDL model with unknown thresholds as the TARDL-n model, where n denotes the number of thresholds. Therefore, (3.13) is the TARDL-1 model and (3.14) the TARDL-2 model.

3.1.1 Testing for One Unknown Threshold

Hansen (1999) proposes the following likelihood ratio test of the null hypothesis of no threshold against the alternative of a single unknown threshold:

$$F_1 = \left(S_0 - S_1 \left(\hat{d} \right) \right) / \hat{\sigma}^2 \tag{3.15}$$

where S_0 and S_1 are the residual sum of squares from (3.12) and (3.13), respectively, and $\hat{\sigma}^2$ the residual variance. Following Hansen (1996, 1999), we propose an easily implemented and asymptotically accurate bootstrapping routine as follows:

(i.) Treat the regressors, $\boldsymbol{x}_t$, the estimated threshold, $\hat{d}$, and the errors, $\hat{\boldsymbol{\epsilon}}_t$, as fixed, and the p initial values of $\boldsymbol{y}$ as given.

(ii.) Draw a $T \times 1$ vector, $\boldsymbol{e}_t^{(b)}$, from the estimated residuals, $\hat{\boldsymbol{\epsilon}}_t$, with replacement.

(ii.) Generate values of $\boldsymbol{y}_t^{(b)}$ under the null hypothesis using (3.12) as the data generating process.

(iv.) Estimate equations (3.12) and (3.13) on the bootstrap sample and compute the associated value of the likelihood ratio statistic (3.15).

(v.) Repeat the process for $b = 1, 2, ..., B$, where B is sufficiently large to provide reliable inference.

3.1.2 Testing for Two Unknown Thresholds

Again following Hansen (*op. cit.*), we propose the following likelihood ratio test of the null hypothesis of one threshold against the alternative hypothesis of two thresholds:

$$F_2 = \frac{S_1\left(\hat{d}\right) - S_2\left(\hat{d}_1,\ \hat{d}_2\right)}{\hat{\sigma}^2} \tag{3.16}$$

where the notation follows obviously from above. This sampling distribution of (3.16) may be evaluated by bootstrapping as follows:

(i.) Treat the regressors, $\boldsymbol{x}_t$, the estimated single threshold, $\hat{d}$, and the errors, $\hat{\boldsymbol{\epsilon}}_t$, as fixed, and the p initial values of $\boldsymbol{y}$ as given.

(ii.) Draw a $T \times 1$ vector, $\boldsymbol{e}_t^{(b)}$, from the estimated residuals of the TARDL-2 model, $\hat{\boldsymbol{\epsilon}}_t$, with replacement.

(ii.) Generate values of $\boldsymbol{y}_t^{(b)}$ under the null hypothesis using the estimated TARDL-1 model as the data generating process.

(iv.) Estimate both equations (3.13) and (3.14) on the bootstrap sample and compute the associated value of the likelihood ratio statistic (3.16).

(v.) Repeat the process for $b = 1, 2, ..., B$, where B is sufficiently large to provide reliable inference.

4 The Phillips Curve in Canada

4.1 The Benchmark Symmetric ARDL Model

Panel (A) of Table 1 presents the results of the benchmark linear (symmetric) ARDL model (3.12) corresponding to the case of no threshold. The results of this simple model are not encouraging. Firstly, the magnitude of the estimated long-run multiplier seems implausible, implying that a 1% change in the rate of unemployment is associated with an 8.67% change in inflation of the opposite sign. It is, therefore, unsurprising that the dynamic multipliers presented in Figure 1 show a clear divergent pattern. Finally, the F_{PSS} statistic takes a value of 4.806 which does not exceed the relevant 5% critical value of 5.73 tabulated by Pesaran, Shin and Smith (2001, p. 300). Hence, we must conclude that no linear long-run levels relationship exists between inflation and unemployment in Canada. This strongly suggests that the linear form of the Phillips curve is profoundly mis-specified in the case of Canada.

4.2 The TARDL-1 Model

Panel (B) of Table 1 presents the results of the TARDL-1 model (3.13). The least squares estimate of the threshold identified by searching over the 70% partial support of Δu_t is -0.19. In this case, we note that the F_{PSS} statistic rejects the null hypothesis at the 5% level and that the long-run multipliers are of a more plausible magnitude than in the linear symmetric case. In particular, we find that the long-run response of inflation to a unit shock in the upper regime (*i.e.* when $\Delta u_t > -0.19$) is -1.68 while in the lower regime it is just -1.10. This suggests that the inflationary 'cost' of reducing unemployment by 1% is higher in an environment in which unemployment is either falling slowly or rising. However, we find little evidence that this difference is statistically significant.

Figure 2 displays the asymmetric cumulative dynamic multipliers derived from the TARDL-1 model. Interestingly, we observe some mild short-run asymmetry that suggests that the inflationary response to unemployment changes is more rapid in the lower regime than in the upper regime. This suggests that the short-run tradeoff between inflation and unemployment may be more acute in the case of rapidly falling unemployment. This is consistent with the idea that falling unemployment levels may provide a rapid demand stimulus that is fed through into the general level of prices with only a minimal lag. It also suggests that economic agents may be more willing to increase their demand than reduce it given a shock of equal size. That is, there may be a behavioural bias at work whereby economic agents respond more rapidly to economic stimuli the more positive the economic outlook.

4.3 The TARDL-2 Model

Searching twice over the grid set consisting of the 70% partial support of Δu_t yields $\hat{d}_1 = -0.12$ and $\hat{d}_2 = 0.12$. The symmetry of these thresholds about zero is purely coincidental; the only restrictions imposed in the grid search routine were that $d_1 < d_2$ and that each regime must contain at least 15% non-zero observations.

Panel (C) of Table 1 presents the results of the TARDL-2 model while Figure 3 presents the associated cumulative dynamic multipliers. Once again, we note that the F_{PSS} test resounding rejects the null hypothesis in this case indicating the existence of a threshold-asymmetric long-run relationship between inflation and output.

Denoting the regimes A, B and C where A is the lower regime (*i.e.* $\Delta u_t < d_1$), B the central or corridor regime (*i.e.* $d_1 < \Delta u_t < d_2$) and C the upper regime (*i.e.* $\Delta u_t > d_2$) we observe a number of interesting asymmetries. Firstly, we find that the long-run multiplier is largest at -1.22 in regime C, smallest and insignificant in regime B and that it takes a moderate value of -0.78 in regime A. This suggests a somewhat enriched version of the explanation offered in the case of the TARDL-1 model above. In this case, the inflationary cost of reducing unemployment by a given amount is greatest in when unemployment is rising rapidly and smallest when it is relatively stable. Indeed, within the range $-0.12 < \Delta u_t < 0.12$ our results suggest that there is in fact no long-run tradeoff between inflation and unemployment. This suggests that if policymakers act gradually while the economy is in this stable central regime and avoid propelling it into either of the outer regimes then they may be able to reduce the rate of unemployment without suffering increasing inflation.

Moving on to the case of short-run asymmetry, we observe a similar pattern to that revealed by the TARDL-1 model in terms of the outer regimes: that is, inflation responds more rapidly to falling unemployment than to rising unemployment. However, we note an interesting effect at work in the central regime where we find a significant *positive* short-run relationship between inflation and unemployment. This suggests that, in the relatively stable central regime, rising unemployment is associated with rising inflation in the short-run and *vice-versa*. This is a striking result which is reminiscent of Eisner's findings (Eisner, 1997).

5 Concluding Remarks

The relationship between unemployment (or aggregate economic activity) and inflation is of central importance to modern macroeconomics as it fundamentally underpins our approach to stabilisation policies including monetary policy. Recent research has suggested that the traditional presumption that the relationship can be well approximated by a simple linear functional form is misled and that, in fact, a range of nonlinearities may exist. We contribute to this literature by investigating a new form of nonlinearity in which the inflation-unemployment tradeoff is nonlinear in relation to the first difference of unemployment. Such a specification has an intuitive appeal as it provides a simple means of modelling the dependence of the inflation-unemployment nexus on the state of the labour market. Moreover, it indirectly accounts for the effect of uncertainty in the sense that rapid changes in the rate of unemployment are likely to signal an uncertain economic outlook in which demand may be rather volatile and the balance of power between employers and employees may become increasingly skewed.

In order to model this form of asymmetry, we generalise the asymmetric ARDL approach developed by SYG to the case of multiple unknown threshold decompositions and develop appropriate estimation and testing routines. On this basis, we find evidence in favour of a three regime model in which the long-run inflation-unemployment tradeoff is relatively acute in the outer regimes (associated with rapid changes in the unemployment rate) but where this is no significant long-run tradeoff in the central (corridor) regime.

This observation has far-reaching ramifications for monetary policy as it suggests that policymakers in Canada may be able to exploit the non-linearity of the Phillips curve to achieve gradual reductions in unemployment without the onset of an inflationary episode. However, policymakers attempting to exploit the stability of the central regime in this manner must be mindful to always act gradually so as to avoid propelling the economy into either of the outer regimes or else the inflationary consequences may be severe. Moroever, our estimates suggest that corridor regime is rather narrow, further underscoring the importance of prudence on the part of opportunistic policymakers.

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(A) Linear Model			(B) TARDL-1 Model			(C) TARDL-2 Model		
Regressor	Coefficient	Std. Error	Regressor	Coefficient	Std. Error	Regressor	Coefficient	Std. Error
Constant	0.206	0.080	Constant	0.374	0.139	Constant	0.683	0.162
π_{t-1}	-0.003	0.007	π_{t-1}	-0.041	0.013	π_{t-1}	-0.073	0.016
u_{t-1}	-0.026	0.009	u^{+}_{t-1}	-0.069	0.017	u^{A}_{t-1}	-0.057	0.013
$\Delta\pi_{t-3}$	0.113	0.046	u^{-}_{t-1}	-0.045	0.011	u^{B}_{t-1}	-0.003	0.062
$\Delta\pi_{t-12}$	-0.449	0.047	$\Delta\pi_{t-3}$	0.104	0.046	u^{C}_{t-1}	-0.089	0.018
Δu_{t-1}	0.180	0.073	$\Delta\pi_{t-5}$	0.121	0.045	$\Delta\pi_{t-2}$	0.111	0.045
Δu_{t-7}	-0.188	0.075	$\Delta\pi_{t-12}$	-0.419	0.046	$\Delta\pi_{t-3}$	0.109	0.045
Δu_{t-11}	-0.292	0.075	Δu^{+}_{t-1}	0.327	0.092	$\Delta\pi_{t-5}$	0.150	0.046
			Δu^{+}_{t-3}	0.265	0.094	$\Delta\pi_{t-12}$	-0.382	0.045
			Δu^{+}_{t-11}	-0.318	0.099	Δu^{A}_{t-2}	0.301	0.132
			Δu^{-}_{t-7}	-0.342	0.136	Δu^{A}_{t-7}	-0.335	0.132
			Δu^{-}_{t-10}	-0.347	0.135	Δu^{B}_{t-1}	1.039	0.342
						Δu^{B}_{t-4}	0.876	0.342
						Δu^{C}_{t-1}	0.327	0.104
						Δu^{C}_{t-3}	0.327	0.107
						Δu^{C}_{t-9}	0.236	0.115
						Δu^{C}_{t-11}	-0.291	0.113
R^2	0.307		R^2	0.315		R^2	0.395	
Adj. R^2	0.292		Adj. R^2	0.298		Adj. R^2	0.365	
F_{PSS}	4.806		F_{PSS}	7.773		F_{PSS}	7.134	

Table 1: Estimation Results

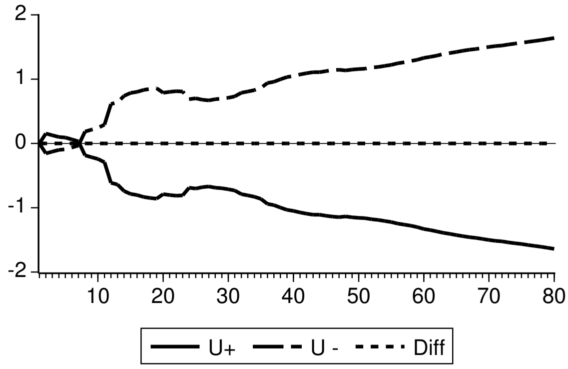

Figure 1: Dynamic Multipliers: Linear Model

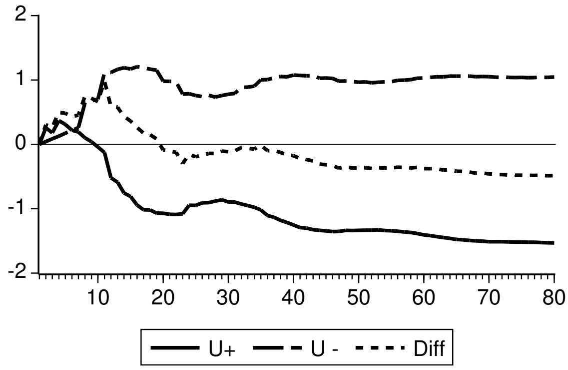

Figure 2: Dynamic Multipliers: TARDL-1 Model ($\hat{d} = -0.19$)

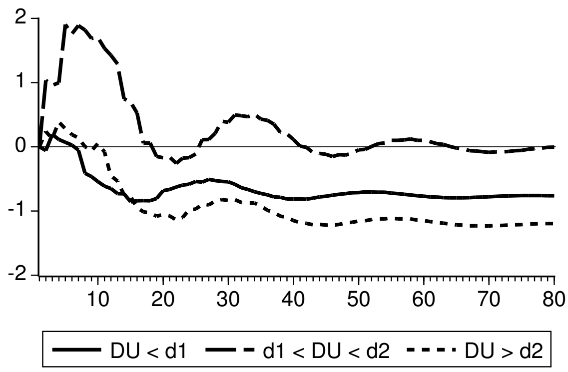

Figure 3: Dynamic Multipliers: TARDL-2 Model ($\hat{d}_1 = -0.12$, $\hat{d}_2 = 0.12$)